

# Mark Scheme (Results)

June 2011

GCE Further Pure FP2 (6668) Paper 1

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## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.

### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\checkmark$  will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper
- $\square$  The second mark is dependent on gaining the first mark

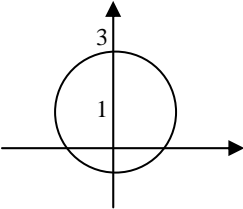
**June 2011**  
**Further Pure Mathematics FP 26668**  
**Mark Scheme**

Question Number	Scheme	Marks
<b>1.</b>	$3x = (x-4)(x+3) \quad x^2 - 4x - 12 = 0$ $x = -2, x = 6$ both Other critical values are $x = -3, x = 0$ $-3 < x < -2, \quad 0 < x < 6$	M1 A1  B1, B1 M1 A1 A1  (7) <b>7</b>
	1 <sup>st</sup> M1 for $\pm(x^2 - 4x - 12) - '=0'$ not required. B marks can be awarded for values appearing in solution e.g. on sketch of graph or in final answer. 2 <sup>nd</sup> M1 for attempt at method using graph sketch or +/- If cvs correct but correct inequalities are not strict award A1A0.	

Question Number	Scheme	Marks
<b>2.</b>  <b>(a)</b>	$\frac{d^3 y}{dx^3} = e^x \left( 2y \frac{d^2 y}{dx^2} + 2 \left( \frac{dy}{dx} \right)^2 + 2y \frac{dy}{dx} \right) + e^x \left( 2y \frac{dy}{dx} + y^2 + 1 \right)$ $\frac{d^3 y}{dx^3} = e^x \left( 2y \frac{d^2 y}{dx^2} + 2 \left( \frac{dy}{dx} \right)^2 + 4y \frac{dy}{dx} + y^2 + 1 \right) \quad (k = 4)$	M1 A1  A1  (3)
<b>(b)</b>	$\left( \frac{d^2 y}{dx^2} \right)_0 = e^0 (4 + 1 + 1) = 6$ $\left( \frac{d^3 y}{dx^3} \right)_0 = e^0 (12 + 8 + 8 + 1 + 1) = 30$ $y = 1 + 2x + \frac{6x^2}{2} + \frac{30x^3}{6} = 1 + 2x + 3x^2 + 5x^3$	B1  B1  M1 A1ft  (4) <b>7</b>
<b>(a)</b>  <b>(b)</b>	1 <sup>st</sup> M1 for evidence of Product Rule 1 <sup>st</sup> A1 for completely correct expression or equivalent 2 <sup>nd</sup> A1 for correct expression or $k = 4$ stated 2 <sup>nd</sup> M1 require four terms and denominators of 2 and 6 (might be implied) A1 follow through from their values in the final answer.	

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} + 5\frac{y}{x} = \frac{\ln x}{x^2} \quad \text{Integrating factor } e^{\int \frac{5}{x}}$ $e^{\int \frac{5}{x}} = e^{5\ln x} = x^5$ $\int x^3 \ln x dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4} dx$ $= \frac{x^4 \ln x}{4} - \frac{x^4}{16} (+C)$ $x^5 y = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C \quad y = \frac{\ln x}{4x} - \frac{1}{16x} + \frac{C}{x^5}$	<p>M1</p> <p>A1</p> <p>M1 M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>(8)</p> <p><b>8</b></p>
	<p>1<sup>st</sup> M1 for attempt at correct Integrating Factor</p> <p>1<sup>st</sup> A1 for simplified IF</p> <p>2<sup>nd</sup> M1 for <math>\frac{\ln x}{x^2}</math> times their IF to give their '<math>x^3 \ln x</math>'</p> <p>3<sup>rd</sup> M1 for attempt at correct Integration by Parts</p> <p>2<sup>nd</sup> A1 for both terms correct</p> <p>3<sup>rd</sup> A1 constant not required</p> <p>4<sup>th</sup> M1 <math>x^5 y = \text{their answer} + C</math></p>	



Question Number	Scheme	Marks
<b>5.</b>		
<b>(a)</b>	$x^2 + (y-1)^2 = 4$	M1 A1 (2)
<b>(b)</b>	 <p>M1: Sketch of circle A1: Evidence of correct centre and radius</p>	M1 A1 (2)
<b>(c)</b>	$w = \frac{(x+iy)+i}{3+i(x+iy)} = \frac{x+i(y+1)}{(3-y)+ix}$ $= \frac{[x+i(y+1)][(3-y)-ix]}{[(3-y)+ix][(3-y)-ix]}$ <p>On <math>x</math>-axis, so imaginary part = 0: <math>(y+1)(3-y) - x^2 = 0</math>  <math>(y+1)(3-y) - x^2 = 0 \Rightarrow x^2 + (y-1)^2 = 4</math>, so <math>Q</math> is on <math>C</math></p>	M1 M1 M1 A1 A1cso (5) <b>9</b>
<b>Alt. (c)</b>	<p>Let <math>w = u + iv</math>: <math>u = \frac{z+i}{3+iz}</math> (since <math>v = 0</math>)</p> $z = \frac{3u-i}{1-ui}$ $z-i = \frac{3u-i-i-u}{1-ui} = \frac{2(u-i)}{1-ui}$ $ z-i  = \frac{2\sqrt{u^2+1}}{\sqrt{u^2+1}} = 2, \text{ so } Q \text{ is on } C$	M1 dM1 M1 A1 A1cso
<b>(a)</b> <b>(b)</b> <b>(c)</b>	<p>M1 Use of <math>z = x + iy</math> and find modulus Award A0 if circle doesn't intersect <math>x</math>-axis twice 1<sup>st</sup> M for subbing <math>z = x + iy</math> and collecting real and imaginary parts 2<sup>nd</sup> M for multiply numerator and denominator by their complex conjugate 3rd M for equating imaginary parts of numerator to 0 Award A1 for equation matching part (a), statement not required.</p>	



Question Number	Scheme	Marks
<p><b>6.</b></p>	$2 + \cos \theta = \frac{5}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\frac{1}{2} \int (2 + \cos \theta)^2 d\theta = \frac{1}{2} \int (4 + 4 \cos \theta + \cos^2 \theta) d\theta$ $= \frac{1}{2} \left[ 4\theta + 4 \sin \theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2} \right]$ <p>Substituting limits <math>\left( \frac{1}{2} \left[ \frac{9\pi}{6} + 4 \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8} \right] = \frac{1}{2} \left( \frac{3\pi}{2} + \frac{17\sqrt{3}}{8} \right) \right)</math></p> <p>Area of triangle = <math>\frac{1}{2} (r \cos \theta)(r \sin \theta) = \frac{1}{2} \times \frac{25}{4} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \left( = \frac{25\sqrt{3}}{32} \right)</math></p> <p>Area of R = <math>\frac{3\pi}{4} + \frac{17\sqrt{3}}{16} - \frac{25\sqrt{3}}{32} = \frac{3\pi}{4} + \frac{9\sqrt{3}}{32}</math></p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(9) <b>9</b></p>
	<p>1<sup>st</sup> M1 for use of <math>\frac{1}{2} \int r^2 d\theta</math> and correct attempt to expand</p> <p>2<sup>nd</sup> M1 for use of double angle formula - <math>\sin 2\theta</math> required in square brackets</p> <p>3<sup>rd</sup> M1 for substituting their limits</p> <p>4<sup>th</sup> M1 for use of <math>\frac{1}{2}</math> base x height</p> <p>5<sup>th</sup> M1 area of sector – area of triangle</p> <p>Please note there are no follow through marks on accuracy.</p>	

Question Number	Scheme	Marks
<b>7.</b>  <b>(a)</b>	$\sin 5\theta = \text{Im}(\cos \theta + i \sin \theta)^5$ $5 \cos^4 \theta (i \sin \theta) + 10 \cos^2 \theta (i^3 \sin^3 \theta) + i^5 \sin^5 \theta$ $= i(5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta)$ $(\text{Im}(\cos \theta + i \sin \theta)^5) = 5 \sin \theta (1 - \sin^2 \theta)^2 - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$ $\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta \quad (*)$	B1 M1 A1 M1 A1cso (5)
<b>(b)</b>	$16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta = 5(3 \sin \theta - 4 \sin^3 \theta)$ $16 \sin^5 \theta - 10 \sin \theta = 0$ $\sin^4 \theta = \frac{5}{8} \quad \theta = 1.095$ <p>Inclusion of solutions from <math>\sin \theta = -\sqrt[4]{\frac{5}{8}}</math></p> <p>Other solutions: <math>\theta = 2.046, 4.237, 5.188</math></p> $\sin \theta = 0 \Rightarrow \theta = 0, \theta = \pi \quad (3.142)$	M1 M1 A1 M1 A1 B1 (6) <b>11</b>
<b>(a)</b>  <b>(b)</b>	<p>Award B if solution considers Imaginary parts and equates to <math>\sin 5\theta</math></p> <p>1<sup>st</sup> M1 for correct attempt at expansion and collection of imaginary parts</p> <p>2<sup>nd</sup> M1 for substitution powers of <math>\cos \theta</math></p> <p>1<sup>st</sup> M for substituting correct expressions</p> <p>2<sup>nd</sup> M for attempting to form equation</p> <p>Imply 3<sup>rd</sup> M if 4.237 or 5.188 seen. Award for their negative root.</p> <p>Ignore <math>2\pi</math> but 2<sup>nd</sup> A0 if other extra solutions given.</p>	

Question Number	Scheme	Marks
<p><b>8.</b></p> <p><b>(a)</b></p>	$m^2 + 6m + 9 = 0 \quad m = -3$ <p>C.F. <math>x = (A + Bt)e^{-3t}</math></p> <p>P.I. <math>x = P \cos 3t + Q \sin 3t</math></p> $\dot{x} = -3P \sin 3t + 3Q \cos 3t$ $\ddot{x} = -9P \cos 3t - 9Q \sin 3t$ $(-9P \cos 3t - 9Q \sin 3t) + 6(-3P \sin 3t + 3Q \cos 3t) + 9(P \cos 3t + Q \sin 3t) = \cos 3t$ $-9P + 18Q + 9P = 1 \quad \text{and} \quad -9Q - 18P + 9Q = 0$ $P = 0 \quad \text{and} \quad Q = \frac{1}{18}$ $x = (A + Bt)e^{-3t} + \frac{1}{18} \sin 3t$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1ft</p> <p>(8)</p>
<p><b>(b)</b></p>	$t = 0: \quad x = A = \frac{1}{2}$ $\ddot{x} = -3(A + Bt)e^{-3t} + Be^{-3t} + \frac{3}{18} \cos 3t$ $t = 0: \quad \ddot{x} = -3A + B + \frac{1}{6} = 0 \quad B = \frac{4}{3}$ $x = \left(\frac{1}{2} + \frac{4t}{3}\right)e^{-3t} + \frac{1}{18} \sin 3t$	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>(5)</p>
<p><b>(c)</b></p>	$t \approx \frac{59\pi}{6} \quad (\approx 30.9)$ $x \approx -\frac{1}{18}$	<p>B1</p> <p>B1ft</p> <p>(2)</p> <p><b>15</b></p>
<p><b>(a)</b></p> <p><b>(b)</b></p>	<p>1<sup>st</sup> M1 Form auxiliary equation and correct attempt to solve. Can be implied from correct exponential.</p> <p>2<sup>nd</sup> M1 for attempt to differentiate PI twice</p> <p>3<sup>rd</sup> M1 for substituting their expression into differential equation</p> <p>4<sup>th</sup> M1 for substitution of both boundary values</p> <p>1<sup>st</sup> M1 for correct attempt to differentiate their answer to part (a)</p> <p>2<sup>nd</sup> M1 for substituting boundary value</p>	





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